

On {wheel, pyramid, theta}-graphs

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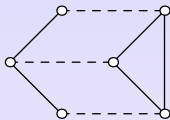
Definitions

Definition

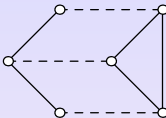
An *induced subgraph* H of a graph G is isomorphic to a graph whose vertex set V_1 is a subset of the vertex set V of G , and whose edge set E_1 consists of all the edges of G with both end vertices in V_1 .

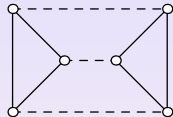
Definitions

- A *pyramid* :



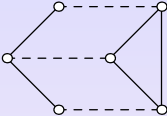
Definitions

- A *pyramid* : A diagram of a pyramid. It consists of a triangular base on the right, with a dashed line extending from its top vertex to the left. From the left end of this dashed line, three solid lines extend upwards and downwards to form a second triangle, representing the pyramid's surface.

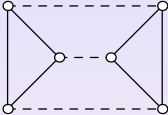
- A *prism* : A diagram of a prism. It consists of two identical triangles, one on the left and one on the right, connected by three vertical lines. A dashed line connects the top vertices of the two triangles, and another dashed line connects the bottom vertices, representing the hidden edges of the prism.

Definitions

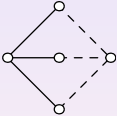
- A *pyramid* :



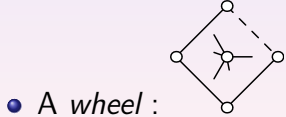
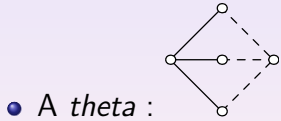
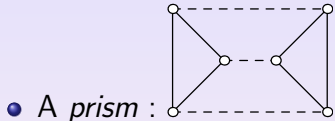
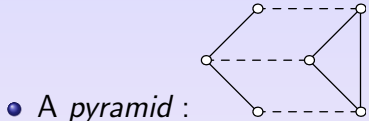
- A *prism* :



- A *theta* :



Definitions



Universally Signable Graphs

- A *Truemper configuration* is a graph isomorphic to a prism, a pyramid, a theta or a wheel

Universally Signable Graphs

- A *Truemper configuration* is a graph isomorphic to a prism, a pyramid, a theta or a wheel
- A graph is *universally signable* if it contains no Truemper configuration as induced subgraph.

Related work.

Theorem (Conforti, Cornuéjols, Kapoor, and Vušković)

If G is a universally signable graph then G is a clique or G is a hole, or G has a clique cutset.

Only Prism

Theorem

If G is a {wheel, theta, pyramid}-free graph, then G is the line graph of a triangle-free chordless graph or G admits a clique cutset.

Only Prism

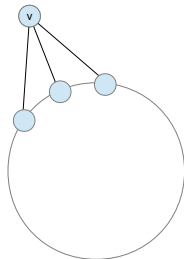
Lemma

Let G be a $\{\text{wheel}, \text{theta}\}$ -free graph. If H a hole in G and $v \in G \setminus H$, then the attachment of v over H is a clique of size at most 2.

Only Prism

Lemma

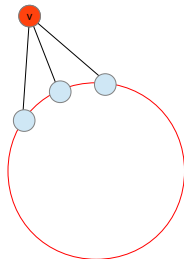
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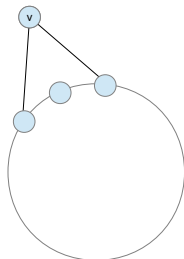
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Only Prism

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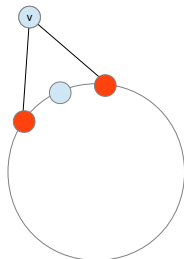
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Only Prism

Lemma

If G is a {wheel, theta, pyramid}-free graph, H is a hole in G , and $x \in V(G \setminus H)$ has a unique neighbor in $V(H)$, then G has a clique cutset.

Related works

Lemma

If a graph G is wheel-free and contains a diamond, then G has a clique cutset.

Theorem (Harary and Holzmann)

If G is a $\{\text{claw, diamond}\}$ -free graph, then G is the line graph of a triangle-free graph.

Main Theorem

Theorem

If G is a {wheel, theta, pyramid}-free graph, then G is the line graph of a triangle-free chordless graph or G admits a clique cutset.

Démonstration.

- If G admits a diamond : Lemma of diamond
- If G admits a claw, then
 - G admits a diamond : contradiction.
 - G admits a hole : By previous lemmas and Harary and Holzmann : G is the line graph of a triangle-free graph R (and cycle of R has a chord, then $G = L(R)$ contains a wheel, so R is chordless).



Future Works

- Others generalizations of the "Universally Signable Graphs"
- Algorithms on these classes (coloration, edge-coloration, clique max, . . .)
- Understanding wheel-free graphs

Thanks