On $\{$wheel, pyramid, theta$\}$-graphs

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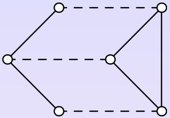
Definitions

Definition

An *induced subgraph* H of a graph G is isomorphic to a graph whose vertex set V1 is a subset of the vertex set V of G, and whose edge set E1 consists of all the edges of G with both end vertices in V1.

Definitions

- A *pyramid* : 

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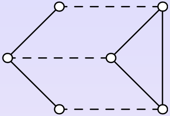
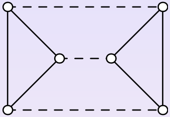
- A *pyramid* : 
- A *prism* : 

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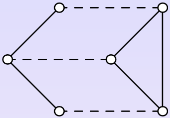
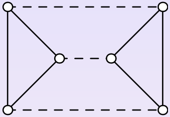
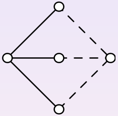
- A *pyramid* : 
- A *prism* : 
- A *theta* : 

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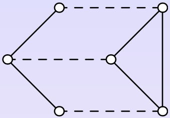
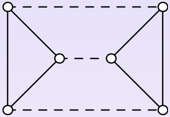
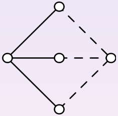
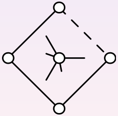
- A *pyramid* : 
- A *prism* : 
- A *theta* : 
- A *wheel* : 

Universally Signable Graphs

- A *Truemper configuration* is a graph isomorphic to a prism, a pyramid, a theta or a wheel

Universally Signable Graphs

- A *Truemper configuration* is a graph isomorphic to a prism, a pyramid, a theta or a wheel
- A graph is *universally signable* if it contains no Truemper configuration as induced subgraph.

Related work.

Theorem (Conforti, Cornuéjols, Kapoor, and Vušković)

If G is a universally signable graph then G is a clique or G is a hole, or G has a clique cutset.

Only Prism

Theorem

If G is a $\{$wheel, theta, pyramid$\}$-free graph, then G is the line graph of a triangle-free chordless graph or G admits a clique cutset.

Only Prism

Lemma

Let G be a $\{$wheel, theta$\}$-free graph. If H a hole in G and $v \in G \setminus H$, then the attachement of v over H is a clique of size at most 2.

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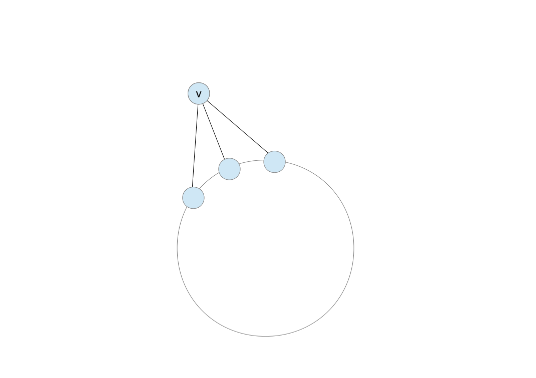

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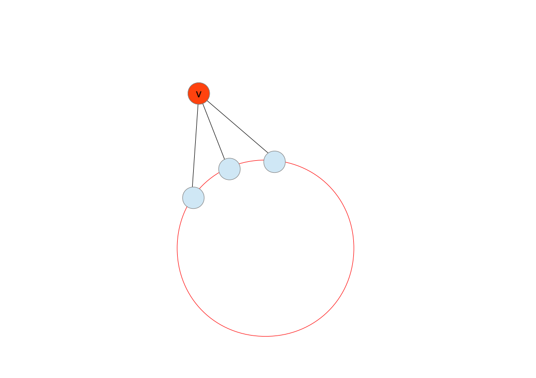

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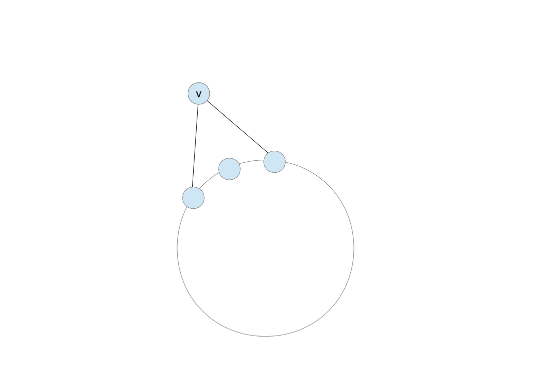

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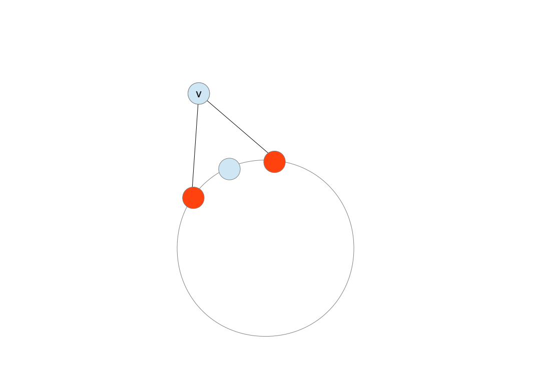

Only Prism

Lemma

If G is a $\{$wheel, theta, pyramid$\}$-free graph, H is a hole in G, and $x \in V(G \setminus H)$ has a unique neighbor in $V(H)$, then G has a clique cutset.

Related works

Lemma

If a graph G is wheel-free and contains a diamond, then G has a clique cutset.

Theorem (Harary and Holzmann)

If G is a $\{$claw, diamond$\}$-free graph, then G is the line graph of a triangle-free graph.

Main Theorem

Theorem

If G is a $\{$wheel, theta, pyramid$\}$-free graph, then G is the line graph of a triangle-free chordless graph or G admits a clique cutset.

Démonstration.

- If G admits a diamond : Lemma of diamond
- If G admits a claw, then
    - G admits a diamond : contradiction.
    - G admits a hole : By previous lemmas and Harary and Holzmann : G is the line graph of a triangle-free graph R (and cycle of R has a chord, then $G = L(R)$ contains a wheel, so R is chordless).

□

Future Works

- Others generalizations of the "Universally Signable Graphs"
- Algorithms on these classes (coloration, edge-coloration, clique max, . . .)
- Understanding wheel-free graphs

Thanks