Local Update of Random Graphs

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Motivations and Setting

Context

- Preservation of the *good properties* of a network under *unpredictable* arrivals and departures of nodes

Our Graph Model

- The network is modelled as a *random* graph G
- When G has n nodes, it has distribution μ_n ($G \sim \mu_n$)

Decentralized Algorithms

- We assume the vertices are only aware of their *local* neighbourhood and the *size* of the network
- They may get knowledge of the network using a **costly** primitive $RandomVertex()$ that returns a node in the network uniformly at random

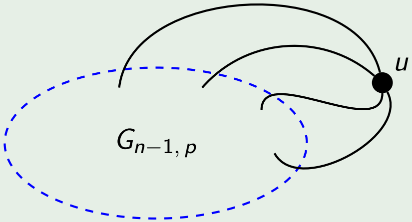
Update Algorithms

Preservation under *arbitrary* update sequences

Given a family of probability distribution $(\mu_n)_{n\in\mathbb{N}}$
(e.g. μ_n is defined over the n-vertices graphs),

- Insertion : If $G \sim \mu_n$ and $u \notin V(G)$ then $\mathcal{A}(G,u) \sim \mu_{n+1}$
- Deletion : If $G \sim \mu_n$ and $u \in V(G)$ then $\mathcal{A}(G,u) \sim \mu_{n-1}$

Example : $G_{n,p}$ (fixed p)

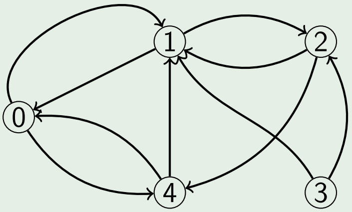


k-out Graphs

Definition

A k-out graph is a simple directed graph with regular out-degree k.

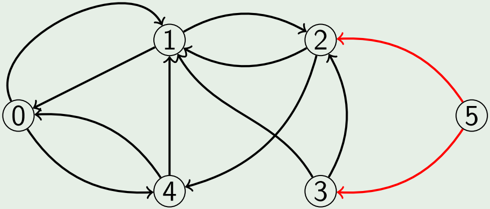
A 2-out graph with 5 vertices



Our Goal: Find update algorithms that preserve the uniform law over k-out graphs.

Insertion Algorithm

Insert vertex 5

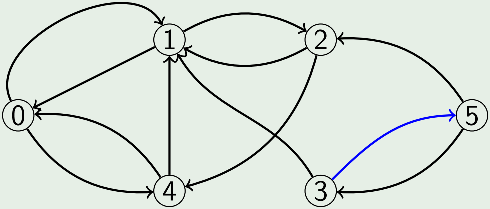


Hum...

Is it enough ?

Insertion Algorithm

Insert vertex 5

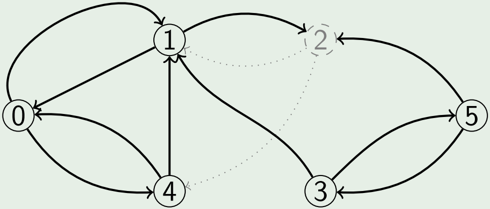


Solution

- Draw $X \sim binomial(n, \frac{k}{n})$, then pick X distinct vertices as your predecessors, and *steal* one edge from each of them.
- Cost: $\lim_{n \to \infty} (\mathbb{E}(RV)) = 2\,k$

Deletion Algorithm

Delete vertex 2



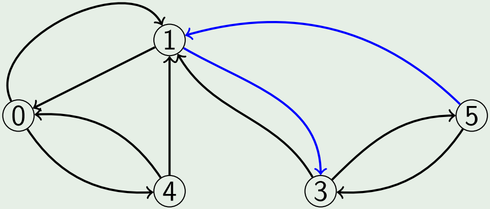
Hum...

If nothing is done the graph is no longer a k-out graph :

- We need to redirect edges!

Deletion Algorithm

Delete vertex 2



Our Algorithms

- Randomly redirect loose edges, Cost : $\lim_{n\to\infty}(\mathbb{E}(RV)) = k$
- Re-use $N^{+}(u)$, Cost : $\lim_{n\to\infty}(\mathbb{E}(RV)) = k \cdot \left(e^{-k} \cdot \frac{k^k}{k!}\right) \simeq \sqrt{\frac{k}{2\pi}}$
- Re-use $N^{-}(u)$ and $N^{+}(u)$, Cost : $\lim_{n\to\infty}(\mathbb{E}(RV)) = 0$

Conclusion (Work in Progress!)

Other considered distributions and extensions

- Some fixed *finite* distribution μ for vertex degrees
- Undirected edges : e.g. pairing model graphs
- Without invariance by vertex renaming

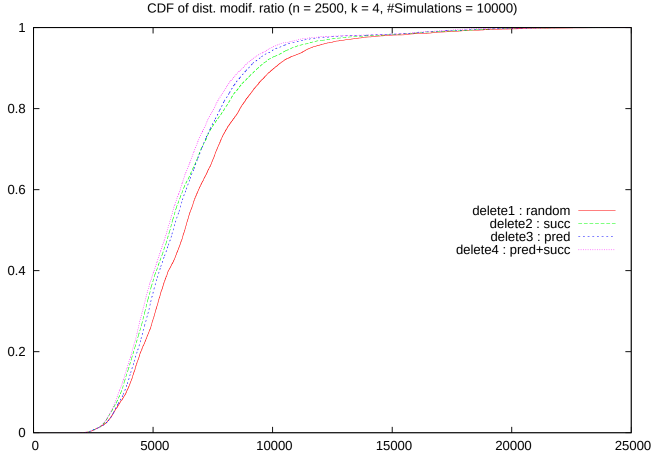
Experimental study

- How the distances (directed and undirected) evolve according to different algorithms (using simulations)
- ...

Merci

Thank you for your attention!

Bonus Frames



Bonus Frames